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A Monte Carlo comparison of LCCA- and ML-based cointegration tests for panel var process with cross-sectional cointegrating vectors[2]

1. INTRODUCTION

Cointegration analysis of non-stationary panel data is typically based on the univariate framework, see Phillips, Moon (1999), where the OLS-based cointegration tests developed firstly by Kao (1999), Pedroni (1999) and McCoskey, Kao (1998) are considered and the DOLS or the FMOLS estimator is employed. Assuming lack of any long-run cross-sectional dependencies, this approach shall work reasonably well for small homogenous systems with one cointegrating vector. However, this will not be valid when the long-run cross-sectional dependencies occurs, for example due to cross-sectional cointegrating vector, see Banerjee et al. (2004) and Jacobson et al. (2008), as well as in case of medium- or large-sized systems with a number of cointegration vectors. Clearly, the panel VAR (PVAR) model proposed by Larsson, Lyhagen (2007) should be considered here, even though in case of lack of long-run cross-sectional dependencies also the global VAR (GVAR) model can be applied, see Pesaran et al. (2004).

The multivariate cointegration analysis of panel data was considered first by Groen, Kleibergen (2003) and Larsson, Lyhagen (2007), who advocated use of VAR models for analysis of non-stationary panel data (see also Larsson, Lyhagen, 2000; Larsson et al., 2001 and Jacobson et al., 2008). Moreover, in the case of panel VAR framework, Anderson et al. (2006) suggested the use of levels canonical correlation analysis (LCCA), as proposed by Box, Tiao (1977) and Bewley et al. (1994), instead of maximum likelihood (ML) estimation proposed by Johansen (1988). Anderson et al. (2006) highlight that they were able to find an additional cross-sectional cointegrating vector in empirical data using Box

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and Tiao approach and the LCCA-based cointegration rank test, as opposed to the results suggested by Johansen's tests.

The aforementioned finding of Anderson et al. (2006) provides a rationale for investigating small sample properties of cointegration rank tests for both Box and Tiao as well as Johansen approach in the framework of panel VAR process with cross-sectional cointegrating vectors. To this end, performance of the min-root and the trace test of Yang, Bewley (1996) are compared with the widely used ML-counterparts derived by Johansen (1988) – the max-root and the trace test.

Since the cointegration rank tests suffer from severe size distortions in small samples, see Johansen (2002), the bootstrap tests are considered. The algorithm of the bootstrap cointegration rank test is similar to algorithms proposed by van Giersbergen (1996) and Svensen (2006), and the asymptotic theory of the bootstrap method was given by Svensen (2006) and Cavaliere et al. (2012). Note however, that even when the bootstrap cointegration rank test or size-corrected test like the test with Bartlett correction is used, some significant size distortions can still occur, especially for high dimensional process such as panel VAR.

The comparative investigations based on the purely time-series context are available in Bewley et al. (1994) and Bewley, Yang (1995). Moreover, performance of canonical correlation estimators of cointegrating vectors for panel VAR models were investigated by Kębłowski (2016).

2. PANEL VAR MODEL, CANONICAL CORRELATION ANALYSIS AND COINTEGRATION TESTS

Consider two main strands of model's specification for the panel VAR/VEC process. The first one would be the straightforward panel augmentation of the VEC model for the double-indexed processes

$$\Delta\mathbf{y}_{it} = \mathbf{\Pi}_i\mathbf{y}_{i,t-1} + \sum_{k=1}^{K-1}\mathbf{\Gamma}_{ki}\Delta\mathbf{y}_{i,t-k} + \mathbf{\Phi}_i\mathbf{d}_t + \boldsymbol{\varepsilon}_{it}, \tag{1}$$

where $\mathbf{y}_{it} = [y_{1it} \;\; y_{2it} \ldots y_{Pit}]'$ is a *P*-dimensional vector of observations for given cross-section i and period t, $\mathbf{\Pi}_i$, and $\mathbf{\Gamma}_{ki}$ are $P \times P$ matrices of coefficient, $\mathbf{d}_t$ and $\mathbf{\Phi}_i$ denote a *N*-dimensional vector of (common) deterministic components and $P \times N$ matrix of their coefficients and $\boldsymbol{\varepsilon}_{it}$ is a *P*-dimensional independently and identically distributed error term with mean equal to zero and covariance matrix $\mathbf{\Omega}_i$ for cross-section i.

Consider next the following VEC model for the panel VAR process

$$\Delta\mathbf{y}_t = \mathbf{\Pi}\mathbf{y}_{t-1} + \sum_{k=1}^{K-1}\mathbf{\Gamma}_k\Delta\mathbf{y}_{t-k} + \mathbf{\Phi}\mathbf{d}_t + \boldsymbol{\varepsilon}_t, \tag{2}$$

where $\mathbf{y}_t = [\mathbf{y}_{1t}' \ \mathbf{y}_{2t}' \dots \mathbf{y}_{It}']'$ is a *IP*-dimensional vector of observations for period t, $\mathbf{\Pi}$ and $\mathbf{\Gamma}_k$ are $IP \times IP$ matrices of coefficient, $\mathbf{\Phi}$ denotes $IP \times N$ matrix of deterministic term coefficients and $\boldsymbol{\varepsilon}_t$ is a *IP*-dimensional independently and identically distributed error term with mean equal to zero and covariance matrix $\mathbf{\Omega}$.

In case of the cointegrated panel VAR process matrix $\mathbf{\Pi}$ can be decomposed into $IP \times IR$ full rank matrices $\mathbf{A}$ and $\mathbf{B}$, and the panel VEC model is

$$\Delta\mathbf{y}_i = \mathbf{A}\mathbf{B}'\mathbf{y}_{t-1} + \sum_{k=1}^{K-1}\mathbf{\Gamma}_k\Delta\mathbf{y}_{t-k} + \mathbf{\Phi}\mathbf{d}_t + \boldsymbol{\varepsilon}_t. \tag{3}$$

Clearly, model (1) assumes lack of any cross-sectional dependencies, whereas model (2) allows for both short- and long-run cross-sectional dependencies, since $\mathbf{A}$, $\mathbf{B}$, $\mathbf{\Gamma}_k$ and $\mathbf{\Omega}$ matrices are not assumed to be block-diagonal (even though in practice $\mathbf{B}$ is most often assumed to be block-diagonal, see Larsson, Lyhagen, 2007). This allows for four different sources of cross-sectional dependence: in the error term, in the short-run dynamics, in the adjustments to the long-run equilibrium and in the cointegration space, as opposed to the assumptions of model (1), and is the main rationale for using model (2) instead of model (1). The only, but significant, disadvantage of using model (2) is the potential dimensionality effect that can limit its application for small samples in case of a large number of cross-sections and variables simultaneously.

The levels canonical correlation analysis of Box and Tiao is performed as follows. At first, the short-run effects are concentrated out and the concentrated regression is

$$\tilde{\mathbf{y}}_t = \mathbf{\Theta}\tilde{\mathbf{y}}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{4}$$

where $\tilde{\mathbf{y}}_{t-1} = \mathbf{y}_t - \mathbf{y}_t\mathbf{z}_t'(\mathbf{z}_t\mathbf{z}_t')^{-1}\mathbf{z}_t$, $\tilde{\mathbf{y}}_{t-1} = \mathbf{y}_{t-1} - \mathbf{y}_{t-1}\mathbf{z}_t'(\mathbf{z}_t\mathbf{z}_t')^{-1}\mathbf{z}_t$ and $\boldsymbol{z}_t = [\Delta\boldsymbol{y}_{t-1}' \ \dots \ \Delta\mathbf{y}_{t-K+1}'\mathbf{d}_t']'$.

Next, the canonical transformation is achieved by solving the eigenvalue problem

$$\left|\lambda\widetilde{\mathbf{Y}}\widetilde{\mathbf{Y}}' - \left(\widetilde{\mathbf{Y}}\widetilde{\mathbf{Y}}_{-1}\right)\left(\widetilde{\mathbf{Y}}_{-1}\widetilde{\mathbf{Y}}_{-1}\right)^{-1}\left(\widetilde{\mathbf{Y}}_{-1}\widetilde{\mathbf{Y}}\right)\right| = 0 \tag{5}$$

for the eigenvalues $0 < \hat{\lambda}_1 < \dots < \hat{\lambda}_{IP} < 1$ and eigenvectors $\widehat{\mathbf{V}} = [\hat{\mathbf{v}}_1 \dots \hat{\mathbf{v}}_{IP}]$, of which the first IR constitute the cointegration subspace.

To compute the cointegration tests of Yang, Bewley (1996), the following series are calculated

$$\hat{\mathbf{y}}_{t-1} = \hat{\mathbf{A}}'_{\perp}\hat{\mathbf{y}}_{t-1} \tag{6a}$$

and

$$\hat{\mathbf{y}}_{t} = \hat{\mathbf{y}}_{t-1} + \hat{\mathbf{A}}'_{\perp}\hat{\mathbf{C}}\mathbf{e}_{t}, \tag{6b}$$

where $\mathbf{e}_t$ denotes residuals from concentrating out the short-run effects in (4) and $\hat{\mathbf{C}}$ is the impact matrix from the moving average representation, $\hat{\mathbf{C}} = \hat{\mathbf{B}}'_{\perp}\left(\hat{\mathbf{A}}_{\perp}'\hat{\boldsymbol{\Gamma}}\hat{\mathbf{B}}_{\perp}\right)^{-1}\hat{\mathbf{A}}_{\perp}$ and $\hat{\boldsymbol{\Gamma}} = \mathbf{I} - \sum_{k=1}^{K-1}\hat{\boldsymbol{\Gamma}}_k$.

Then the following eigenvalue problem is solved

$$\left|\mu\tilde{\mathbf{Y}}\tilde{\mathbf{Y}}' - \left(\tilde{\mathbf{Y}}\tilde{\mathbf{Y}}_{-1}'\right)\left(\tilde{\mathbf{Y}}_{-1}\tilde{\mathbf{Y}}_{-1}'\right)^{-1}\left(\tilde{\mathbf{Y}}_{-1}\tilde{\mathbf{Y}}'\right)\right| = 0 \tag{7}$$

for the eigenvalues $0 < \hat{\mu}_1 < \cdots < \hat{\mu}_{I(P-R)} < 1$. The LCCA-based cointegration rank tests of the null $H_0\colon \mathrm{rank}\,(\boldsymbol{\Pi}) = IR$ vs. the alternative $H_1\colon \mathrm{rank}\,(\boldsymbol{\Pi}) = IR$ are calculated as the minimum-root test $minroot^{LCCA} = T(1-\hat{\mu}_1)$ and the trace test $trace^{LCCA} = T\sum_{j=1}^{I(P-R)}\left(1-\hat{\mu}_j\right)$. Both tests diverge under the alternative, see Yang, Bewley (1996).

Similarly, the canonical correlation analysis of differences and lagged levels, derived by Johansen (1988), is calculated by concentrating out at first the short-run effects, thus the concentrated regression is

$$\Delta\hat{\mathbf{y}}_t = \mathbf{A}\mathbf{B}'\tilde{\mathbf{y}}_{t-1}. \tag{8}$$

Then the canonical transformation is performed by solving

$$\left|\lambda\tilde{\mathbf{Y}}_{-1}\tilde{\mathbf{Y}}_{-1}{}' - \left(\tilde{\mathbf{Y}}_{-1}\Delta\tilde{\mathbf{Y}}'\right)\left(\Delta\tilde{\mathbf{Y}}\Delta\tilde{\mathbf{Y}}'\right)^{-1}\left(\Delta\tilde{\mathbf{Y}}\tilde{\mathbf{Y}}_{-1}{}'\right)\right| = 0 \tag{9}$$

for the eigenvalues $1 < \hat{\lambda}_1 < \cdots < \hat{\lambda}_{IP} < 0$ and $\hat{\mathbf{B}} = [\hat{\mathbf{v}}_1 \ldots \hat{\mathbf{v}}_{IR}]$. The cointegration tests of the same hypotheses as in Yang, Bewley (1996) are calculated as the maximum-root test $maxroot^{LCCA} = -T\ln\left(1-\hat{\lambda}_{IR+1}\right)$ and the trace test $race = -T\sum_{j=IR+1}^{IP}\ln\left(1-\hat{\lambda}_j\right)$.

3. DESIGN OF EXPERIMENT AND RESULTS

Monte Carlo simulation is used to compare performance of the LCCA-based and the ML-based cointegration rank tests within the framework of second-order panel VEC model (3) with five variables for each cross section $(P = 5)$ and a constant restricted to the cointegration space

$$\Delta\hat{\mathbf{y}}_t = \mathbf{AB}'[\mathbf{y}'_{t-1}\mathbf{j}']' + \mathbf{\Gamma}_1\Delta\mathbf{y}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{10}$$

where short- and long-run cross-sectional dependencies are allowed and the error term $\boldsymbol{\varepsilon}_t$ comes from the multivariate normal distribution, $\boldsymbol{\varepsilon}_t \backsim N_{IP}(\mathbf{0};\mathbf{\Omega})$, with covariance matrix from the inverse Wishart distribution – $\backsim W_{IP}^{-1}(\mathbf{I};100)$.

The results for the case with an unrestricted constant are not reported here (available upon request), since they do not alter the conclusions drawn from the former case. Two cointegrating vectors for each cross-section are imposed plus an additional cross-sectional cointegrating vector describing a homogeneous long-run relationship between the fifth variable of each cross-section

$$\mathbf{B} = \begin{bmatrix} \begin{matrix}1-10\,00\\0\,0\,1-10\end{matrix} & \mathbf{0} & \cdots & \mathbf{0} & \begin{matrix}1\\1\end{matrix} \\ \mathbf{0} & \begin{matrix}1-10\,0\,0\\0\,0\,1-10\end{matrix} & \cdots & \mathbf{0} & \begin{matrix}1\\1\end{matrix} \\ \vdots & \vdots & & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \begin{matrix}1-10\,00\\0\,01-10\end{matrix} & \begin{matrix}1\\1\end{matrix} \\ 0\,0\,0\,0\,1 & 0\,0\,0\,0\,\beta & \cdots & 0\,0\,0\,0\,\beta & 1 \end{bmatrix}, \tag{11}$$

where $\beta = -\left(1/(I-1)\right)$. The loadings matrix allows for the long-run cross--sectional adjustments

$$\mathbf{A} = \begin{bmatrix} -0.5 & 0 & 0 & 0 & 0 & \dots & -0.1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.5 & 0 & 0 & & 0 & 0 & -0.1 & 0 & 0 \\ & & \vdots & & & & & & \vdots & & \\ -0.1 & 0 & 0 & 0 & 0 & \dots & -0.5 & 0 & 0 & 0 & 0 \\ 0 & 0 & -0.1 & 0 & 0 & & 0 & 0 & -0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & -0.5 & \dots & 0 & 0 & 0 & 0 & -0.5 \end{bmatrix}. \tag{12}$$

The cross-sectional dependence in the short-run adjustments is allowed, since $\underset{i=j}{\forall}\gamma_{ij,1} = 0.5$ and $\underset{i\neq j}{\forall}\gamma_{ij,1} \sim U(-0.1,0.1)$. The roots of the autoregressive polynomial are computed in order to exclude explosive roots in the DGP. The initial values comes from the multivariate normal distribution and the first hundred observations are removed. The number of cross-sections varies from two to six and the sample size is $T \in \{100, 200, 400, 800\}$. The number of replications for the Monte Carlo simulation is 10000.

The algorithm of the bootstrap method for cointegration rank tests is similar to algorithms proposed by van Giersbergen (1996) and Svensen (2006), and it is as follows:

Step 1. Estimate (2) by means of LCCA, for $minroot^{LCCA}$ and $trace^{LCCA}$ tests, or ML for $maxroot$ and $trace$ tests, and calculate a realization of the test statistic.

Step 2. Estimate the parameters under the null – (3) and check whether the roots of the autoregressive polynomial are equal to 1 or lie outside the unit circle.
Step 3. Using (3) compute recursively bootstrap sample using sampled residuals drawn with replacement from the estimated residuals.
Step 4. Calculate bootstrap realization of the test statistic.
Step 5. Repeat steps 3 to 4 a large number of times.
Step 6. Reject the null $H_0\colon \mathrm{rank}\,(\mathbf{\Pi}) = IR$ if the test statistic from step (1) is larger than the critical value from the bootstrap distribution.
The number of replications for the bootstrap method is 1000.

The relative frequencies of rejecting the false null $H_0\colon \mathrm{rank}\,(\mathbf{\Pi}) = IR$ and the true null $H_0\colon \mathrm{rank}\,(\mathbf{\Pi}) = IR + 1$ of bootstrap cointegration tests for PVAR model (10) are contained in tables 1–2 respectively. Comparison of the empirical size of the tests reveals that the ML-based cointegration rank tests $maxroot$ and $trace$ are undersized in small samples, as compared to 5% nominal size, whereas the LCCA-based cointegration rank tests $minroot^{LCCA}$ and $trace^{LCCA}$ are oversized. The size distortion becomes negligible in general only for a quite long samples of at least 400 observations within time dimension, even though for PVAR models with moderate dimension $(IP \leq 20)$ and 200 observations it seems to be still under control.

Table 1. EMPIRICAL SIZE OF BOOTSTRAP COINTEGRATION TESTS FOR PANEL VAR PROCESS WITH CROSS-SECTIONAL COINTEGRATING VECTOR

I	*T*	ML		LCCA	
		maxroot	*trace*	*minroot*	*trace*
2	100	0.025	0.045	0.124	0.134
	200	0.058	0.081	0.065	0.057
	400	0.057	0.088	0.049	0.037
	800	0.056	0.053	0.043	0.033
3	100	0.000	0.008	0.306	0.308
	200	0.046	0.050	0.108	0.082
	400	0.057	0.056	0.052	0.039
	800	0.052	0.064	0.036	0.020
4	100	0.000	0.001	0.240	0.777
	200	0.021	0.036	0.090	0.079
	400	0.040	0.057	0.061	0.039
	800	0.048	0.048	0.029	0.015
5	100	0.000	0.000	0.334	0.959
	200	0.000	0.012	0.217	0.226
	400	0.052	0.064	0.066	0.052
	800	0.066	0.073	0.054	0.030
6	100	—	—	—	—
	200	0.000	0.002	0.225	0.557
	400	0.049	0.065	0.075	0.065
	800	0.048	0.063	0.046	0.026

Table 2. PERFORMANCE OF BOOTSTRAP COINTEGRATION TESTS FOR PANEL VAR PROCESS WITH CROSS-SECTIONAL COINTEGRATING VECTOR

I	*T*	ML		LCCA	
		maxroot	*trace*	*minroot*	*trace*
2	100	0.540	0.432	0.280	0.320
	200	1.000	0.999	0.619	0.495
	400	1.000	1.000	1.000	1.000
	800	1.000	1.000	1.000	1.000
3	100	0.002	0.026	0.447	0.540
	200	0.948	0.735	0.431	0.486
	400	1.000	1.000	0.987	0.965
	800	1.000	1.000	0.997	0.987
4	100	0.000	0.002	0.354	0.875
	200	0.298	0.293	0.605	0.559
	400	1.000	0.999	0.989	0.922
	800	1.000	1.000	0.992	0.957
5	100	0.000	0.000	0.386	0.969
	200	0.004	0.074	0.798	0.740
	400	1.000	0.915	0.990	0.899
	800	1.000	1.000	0.998	0.955
6	100	—	—	—	—
	200	0.000	0.017	0.546	0.871
	400	0.990	0.638	0.992	0.880
	800	1.000	1.000	0.993	0.931

With respect to performance of both group of cointegration rank tests it can be seen that both the ML-based approach as well as the LCCA-based approach perform very poorly in short samples, as compared to the actual size of each test 0.05. According to the results, it is very likely that in a very short samples of 100 observations in time dimension and less, the LCCA-based cointegration rank tests will indicate at an additional cross-sectional cointegrating vector, the ML-based counterparts will lead to an opposite conclusion, all this regardless of the actual existence of the cross-sectional cointegrating vector.

Moreover, even though the performance of the tests is not size-adjusted, it can be easily noted that the ML-based cointegration rank tests perform in general better than their LCCA-based counterparts in detecting an additional cross-sectional cointegrating vector within a PVAR framework, see the results for $T = 200$ and $I = 2, 3$ for example. As expected, in the case of long panel data with 400 observations within time dimension and more, performance of both approaches is close to unity and thus comparable.

The results of Monte Carlo investigation on small sample properties of bootstrap cointegration rank tests for the PVAR model clearly suggest that the tests based on levels canonical correlation analysis are in general outperformed by the maximum likelihood cointegration rank tests, if the dynamic properties of the underlying process are properly specified. Moreover, the higher the number of cross-sections is, the more observations within time dimension is needed in order to unambiguously infer on the cointegration rank for the PVAR model. If the dimension of the PVAR model IP exceeds 15 then as many as 400 observations can be needed in order to properly identify an additional cross-sectional cointegrating vector using the ML-based approach.

Clearly, the results show that the application of the bootstrap panel cointegrating rank test for the PVAR model given by (2) is in practice limited to the panels with few cross-sections, which makes the application of the tests for macroeconomic panels with larger cross-sectional dimension impossible in fact. This is an important drawback of the approach based on the PVAR model.

4. CONCLUSIONS

In this paper we have examined small sample properties of the bootstrap cointegration rank tests for the unrestricted panel VAR model when short- and long-run cross-sectional dependencies occur in the underlying process generating non-stationary panel data. Two basic frameworks were employed: levels canonical correlation analysis and maximum likelihood estimation. The results shows that the bootstrap cointegration rank tests for the panel VAR model suffer from severe size distortions in small samples, with downward bias of the ML-based tests and the upward bias of the LCCA-based tests. Weak performance of both approaches is observed for a very short sample of about 100 observations within time dimension. As a result, the LCCA-based cointegration rank tests will easily indicate at an additional cross-sectional cointegrating vector, the ML-based counterparts can lead to an opposite conclusion, all this regardless of the actual existence of the additional cointegrating vector. Moreover, it was found that the bootstrap cointegration rank tests for panel VAR model based on levels canonical correlation analysis are in general outperformed by the maximum likelihood cointegration rank tests.

The results of the investigation indicate that the bootstrap ML-based cointegration rank tests perform quite well with respect to size distortion and power for small- and medium-sized PVAR models $(IP < 20)$, if there are at least 200 observations within time dimension. Whereas in case of large-sized PVAR models, say $(IP \geq 20)$, long panel data with about 400 observations within time dimension are necessary.

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ANALIZA MONTE CARLO WŁASNOŚCI TESTÓW KOINTEGRACJI DLA PANELOWEGO PROCESU VAR Z MIĘDZYPRZEKROJOWYMI WEKTORAMI KOINTEGRUJĄCYMI

Streszczenie

W artykule przedstawiono wyniki badania własności bootstrapowych testów kointegracji dla panelowego procesu VAR z międzyprzekrojowymi wektorami kointegrującymi. Wyniki badania wskazują, że bootstrapowe testy kointegracji dla modelu PVAR, które oparte są na analizie korelacji kanonicznej poziomów, cechują się przeszacowaniem rozmiaru testu, z kolei bootstrapowe testy kointegracji dla modelu PVAR wywiedzione z metody największej wiarygodności charakteryzują się zwykle niedoszacowaniem rozmiaru testu. Wykazano również, że bootstrapowe testy kointegracji dla modelu PVAR wywiedzione z metody największej wiarygodności cechują się zwykle lepszymi własnościami ze względu na moc testu. Wyniki badania wskazują, że własności bootstrapowych testów kointegracji dla modelu PVAR wywiedzionych z metody największej wiarygodności cechują się satysfakcjonującymi własnościami małopróbkowymi dla małowymiarowych modeli PVAR z ograniczoną liczbą przekroi.

Słowa kluczowe: międzyprzekrojowe wektory kointegrujące, analiza korelacji kanonicznej, testy kointegracji, panelowy model VAR, procedura Boxa i Tiao

A MONTE CARLO COMPARISON OF LCCA- AND ML-BASED COINTEGRATION TESTS FOR PANEL VAR PROCESS WITH CROSS-SECTIONAL COINTEGRATING VECTORS

Abstract

Small-sample properties of bootstrap cointegration rank tests for unrestricted panel VAR process are considered when long-run cross-sectional dependencies occur. It is shown that the bootstrap cointegration rank tests for the panel VAR model based on levels canonical correlation analysis are oversized, whereas the bootstrap cointegration rank tests based on maximum likelihood framework are undersized. Moreover, the former tests are in general outperformed by the latter in terms of performance. The results of the investigation indicate that the ML-based bootstrap cointegration rank tests perform well in small samples for small-sized panel VAR models with a few cross-sections.

Keywords: cross-sectional cointegrating vectors, canonical correlation analysis, cointegration tests, panel VAR model, Box and Tiao approach